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**CALCULATIONS OF SINGLE-INCLUSIVE CROSS  
SECTIONS AND SPIN ASYMMETRIES IN  $pp$   
SCATTERING\***

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We present calculations of cross sections and spin asymmetries in single-inclusive reactions in  $pp$  scattering. We discuss next-to-leading order predictions as well as all-order soft-gluon threshold resummations.

**1. Introduction**

Single-inclusive reactions in  $pp$  scattering, such as  $pp \rightarrow \gamma X$ ,  $pp \rightarrow \pi X$ ,  $pp \rightarrow \text{jet } X$ , play an important role in QCD. At sufficiently large produced transverse momentum,  $p_T$ , QCD perturbation theory (pQCD) can be used to derive predictions for these reactions. Since high  $p_T$  implies large momentum transfer, the cross section may be factorized at leading power in  $p_T$  into convolutions of long-distance pieces representing the structure of the initial hadrons, and parts that are short-distance and describe the hard interactions of the partons. The long-distance contributions are universal, that is, they are the same in any inelastic reaction, whereas the short-distance pieces depend only on large scales and, therefore, can be evaluated using QCD perturbation theory. Because of this, single-inclusive cross sections offer unique possibilities to probe the structure of the initial hadrons in ways that are complementary to deeply-inelastic scattering. At the same time, they test the perturbative framework, for example, the relevance of higher orders in the perturbative expansion and of power-suppressed contributions to the cross section.

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Of special interest is the case when the initial protons are polarized. At RHIC, one measures spin asymmetries for single-inclusive reactions, in order to investigate the spin structure of the nucleon<sup>1</sup>. A particular focus here is on the gluon polarization in the nucleon,  $\Delta g \equiv g^\uparrow - g^\downarrow$ .

In the following, we will present some theoretical predictions for cross sections and spin asymmetries for single-inclusive reactions. We will first discuss the double-longitudinal spin asymmetries  $A_{\text{LL}}$  for pion and jet production at RHIC and their sensitivities to  $\Delta g$ <sup>2,3</sup>. In the second part, we will give results for new calculations<sup>4</sup> of the unpolarized cross section for  $pp \rightarrow \pi^0 X$  in the fixed-target regime, which show a greatly improved description of the available experimental data.

## 2. Spin asymmetries for $pp \rightarrow (\pi^0, \text{jet}) X$ at RHIC

We consider the double-spin asymmetry

$$A_{\text{LL}} \equiv \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} \equiv \frac{d\Delta\sigma}{d\sigma} , \quad (1)$$

where the superscripts denote the helicities of the initial protons. According to the factorization theorem the spin-dependent cross section  $\Delta\sigma$  can be written in terms of the spin-dependent parton distributions  $\Delta f$  as

$$\frac{d\Delta\sigma}{dp_T d\eta} = \sum_{a,b} \Delta f_a(x_a, \mu) \otimes \Delta f_b(x_b, \mu) \otimes \frac{d\Delta\hat{\sigma}_{ab}}{dp_T d\eta}(x_a, x_b, p_T, \eta, \mu) , \quad (2)$$

where the symbols  $\otimes$  denote convolutions and where the sum is over all contributing partonic channels. We have written Eq. (2) for the case of jet production; for pion production there is an additional convolution with a pion fragmentation function. As mentioned above, the parton-level cross sections may be evaluated in QCD perturbation theory:

$$d\Delta\hat{\sigma}_{ab} = d\Delta\hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s}{\pi} d\Delta\hat{\sigma}_{ab}^{(1)} + \dots , \quad (3)$$

corresponding to “leading order” (LO), “next-to-leading order” (NLO), and so forth. The NLO corrections for the spin-dependent cross sections for inclusive-hadron and jet production were published in<sup>2,5</sup> and<sup>3,6</sup>, respectively. They are crucial for making reliable quantitative predictions and for analyzing the forthcoming RHIC data in terms of spin-dependent parton densities. The corrections can be sizable and they reduce the dependence on the factorization/renormalization scale  $\mu$  in Eq. (2). In case of jet production, NLO corrections are also of particular importance since it is only at NLO that the QCD structure of the jet starts to play a role.

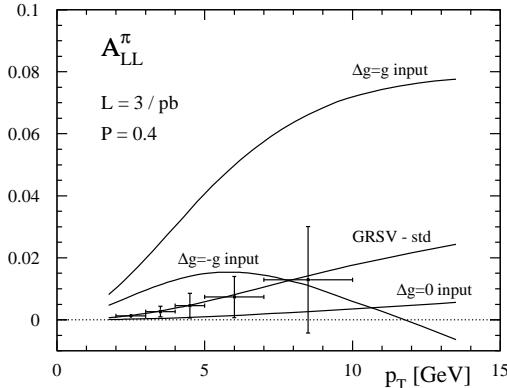


Figure 1. NLO spin asymmetry  $A_{LL}^{\pi}$  for  $\pi^0$  production, using several GRSV polarized parton densities<sup>7</sup> with different gluon polarizations.

Figure 1 shows NLO predictions for the spin asymmetry  $A_{LL}$  for high- $p_T$  pion production for collisions at  $\sqrt{S} = 200$  GeV at RHIC. We have used various sets of polarized parton densities of<sup>7</sup>, which mainly differ in  $\Delta g$ . As one can see, the spin asymmetry strongly depends on  $\Delta g$ , so that measurements of  $A_{LL}$  at RHIC should give direct and clear information. The “error bars” in the figure are uncertainties expected for measurements with an integrated luminosity of 3/pb and beam polarization  $P=0.4$ . We note that PHENIX has already presented preliminary data<sup>8</sup> for  $A_{LL}$ . We also mention that the figure shows that at lower  $p_T$  the asymmetry is not sensitive to the *sign* of  $\Delta g$ . This is related to the dominance of the  $gg$  scattering channel which is approximately quadratic in  $\Delta g$ . In fact it can be shown that  $A_{LL}$  in leading-power QCD can hardly be negative at  $p_T$  of a few GeV<sup>9</sup>. One may obtain better sensitivity to the sign of  $\Delta g$  by expanding kinematics to the forward rapidity region.

Figure 2 shows predictions for the spin asymmetry  $A_{LL}$  for high- $p_T$  jet production. The gross features are rather similar to the pion asymmetry, except that everything is shifted by roughly a factor two in  $p_T$ . This is due to the fact that a pion takes only a certain fraction of  $\sim \mathcal{O}(50\%)$  of the outgoing parton’s momentum, so that the hard scattering took place at roughly twice the pion transverse momentum. A jet, however, will carry the full transverse momentum of a produced parton.

We emphasize that PHENIX and STAR have presented measurements<sup>10</sup> of the unpolarized cross section for  $pp \rightarrow \pi^0 X$ . These are well described by the corresponding NLO QCD calculations<sup>2,5</sup>, providing con-

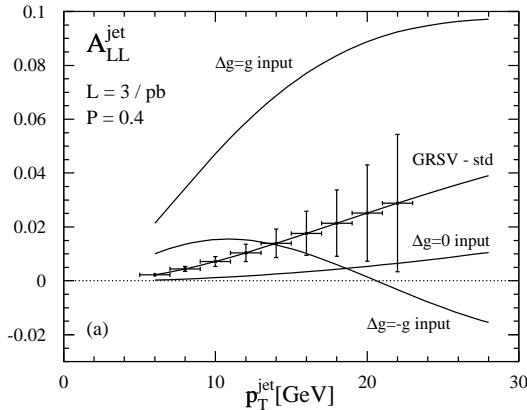


Figure 2. Same as Fig. 1, but for inclusive jet production <sup>3</sup> at RHIC.

fidence that the NLO pQCD hard-scattering framework is indeed adequate in the RHIC domain. This is in contrast to what was found in comparisons <sup>11</sup> between NLO theory and data for inclusive-hadron production taken in the fixed-target regime. We will turn to this issue next.

### 3. Threshold resummation for inclusive-hadron production

One may further improve the theoretical calculations by an all-order resummation of large logarithmic corrections to the partonic cross sections <sup>4</sup>. At partonic threshold, when the initial partons have just enough energy to produce a high-transverse momentum parton (which subsequently fragments into the observed pion) and a massless recoiling jet, the phase space available for gluon bremsstrahlung vanishes, resulting in large logarithmic corrections to the partonic cross section. For the rapidity-integrated cross section, partonic threshold is reached when  $\hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}} = 1$ , where  $\sqrt{\hat{s}}$  is the partonic center-of-mass (c.m.) energy, and  $\hat{p}_T$  is the transverse momentum of the produced parton fragmenting into the hadron. The leading large contributions near threshold arise as  $\alpha_s^k \ln^{2k} (1 - \hat{x}_T^2)$  at the  $k$ th order in perturbation theory. Sufficiently close to threshold, the perturbative series will be only useful if such terms are taken into account to all orders in  $\alpha_s$ , which is achieved by threshold resummation <sup>12</sup>. This resummation has been derived for a number of cases of interest, to next-to-leading logarithmic (NLL) order, in particular also for jet production <sup>13</sup> which proceeds through the same partonic channels as inclusive-hadron production.

The larger  $\hat{x}_T$ , the more dominant the threshold logarithms will be. Since  $\hat{s} = x_a x_b S$ , where  $x_{a,b}$  are the partonic momentum fractions and  $\sqrt{S}$  is the hadronic c.m. energy, and since the parton distribution functions fall rapidly with increasing  $x_{a,b}$ , threshold effects become more and more relevant as the hadronic scaling variable  $x_T \equiv 2p_T/\sqrt{S}$  goes to one. This means that the fixed-target regime with  $3 \text{ GeV} \lesssim p_T \lesssim 10 \text{ GeV}$  and  $\sqrt{S}$  of  $20\text{--}30 \text{ GeV}$  is the place where threshold resummations are expected to be particularly relevant and useful.

The resummation is performed in Mellin- $N$  moment space, where the logarithms  $\alpha_s^k \ln^{2k} (1 - \hat{x}_T^2)$  turn into  $\alpha_s^k \ln^{2k}(N)$ , which then exponentiate. For inclusive-hadron production, because of the color-structure of the underlying Born  $2 \rightarrow 2$  QCD processes, one actually obtains a *sum* of exponentials in the resummed expression. Details may be found in <sup>4</sup>. Here, we only give a brief indication of the qualitative effects resulting from resummation. For a given partonic channel  $ab \rightarrow cd$ , the leading logarithms exponentiate in  $N$  space as

$$\hat{\sigma}_{ab \rightarrow cd}^{(res)}(N) \propto \exp \left[ \frac{\alpha_s}{\pi} \left( C_a + C_b + C_c - \frac{1}{2} C_d \right) \ln^2(N) \right], \quad (4)$$

where

$$C_g = C_A = N_c = 3, \quad C_q = C_F = (N_c^2 - 1)/2N_c = 4/3. \quad (5)$$

This exponent is clearly positive for each of the partonic channels, which means that the soft-gluon effects will lead to an enhancement of the cross section. Indeed, as may be seen from Fig. 3, resummation dramatically increases the cross section in the fixed-target regime. The example we give is a comparison of NLO and NLL resummed predictions at  $\sqrt{S} = 31.5 \text{ GeV}$  with the data of E706 <sup>14</sup> at that energy. We have used the “KKP” set of pion fragmentation functions <sup>15</sup>, and the parton distributions of <sup>16</sup>. We finally note that the results shown in Fig. 3 are also interesting with respect to the size of power corrections to the cross section. Resummation may actually suggest the structure of nonperturbative power corrections. For a recent study of this for single-inclusive cross sections, see <sup>17</sup>.

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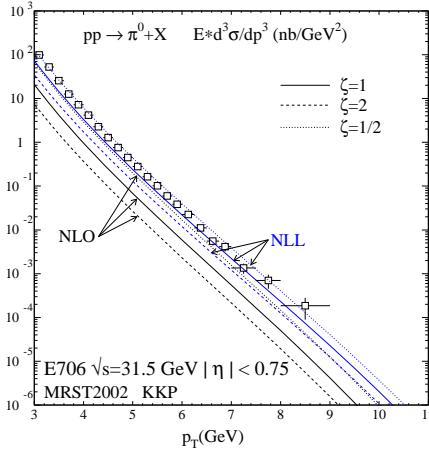


Figure 3. NLO and NLL resummed<sup>4</sup> results for the cross section for  $pp \rightarrow \pi^0 X$  for E706 kinematics. Results are given for three different choices of scales,  $\mu = \zeta p_T$ , where  $\zeta = 1/2, 1, 2$ . Data are from<sup>14</sup>.

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